Arfken 6.4.4.

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16:33

The problem is to evaluate the integral $\oint_C \frac{dz}{(z^2-1)}$ Where C is the circle $|z| = 2$ that is a circle with radius 2 centered on origo in the complex plane.

Observe that this is a contour integral around a closed curve in the complex plane with a singularity at z = 1

This can be seen by expansion of the denominator with the conjugate rule.

$$\oint_C \frac{dz}{z^2-1} = \oint_C \frac{dz}{(z-1)(z+1)} = \oint_C \frac{\frac{dz}{z+1}}{z-1}$$

Cauchy's integralformula

$$\oint_C \frac{f(z)\,dz}{(z-a)} = 2\pi i f(a)$$

$$\oint_C \frac{dz}{z^2-1} = 2\pi i f(1) = 2\pi i \cdot \frac{1}{2} = \pi i$$

Answer: $\oint_{|z|=2} \frac{dz}{z^2-1} = \pi i$